Pour $x \leq 0$: $d(x) = f(x) - y = \dfrac{x}{2\sqrt{x^2+1}} - \dfrac{x}{2} = \dfrac{-x\sqrt{x^2+1}}{2\sqrt{x^2+1}}$

$$= \frac{x\left(1-\sqrt{x^2+1}\right)}{2\sqrt{x^2+1}} = \frac{x\left(1-\sqrt{x^2+1}\right)\left(1+\sqrt{x^2+1}\right)}{2\sqrt{x^2+1}(1+)\sqrt{x^2+1}}$$

$$= \frac{-x^3}{2\sqrt{x^2+1}\left(1+\sqrt{x^2+1}\right)} \geq 0$$

Cf est au dessus de D sur] - ∞ , 0] .

4°/ pour $x > 0$:

$$f'(x) = \frac{\dfrac{x^2}{\sqrt{x^2+1}} - \left(\sqrt{x^2+1}-1\right)}{x^2} = \frac{x^2 - \left(x^2+1-\sqrt{x^2+1}\right)}{x^2\sqrt{x^2+1}}$$

$$= \frac{\sqrt{x^2+1}-1}{x^2\sqrt{x^2+1}} = \frac{\left(\sqrt{x^2+1}-1\right)\left(\sqrt{x^2+1}-1\right)}{x^2\sqrt{x^2+1}}$$

$$= \frac{1}{\sqrt{x^2+1}\left(\sqrt{x^2+1}+1\right)}$$

Pour $x < 0$: $f'(x) = \dfrac{2\sqrt{x^2+1} - 2x\dfrac{x}{\sqrt{x^2+1}}}{4\left(x^2+1\right)} = \dfrac{1}{2\left(x^2+1\right)\sqrt{x^2+1}}$

On a pour tout $x \in \mathbb{R}$: $f'(x) > 0$

x	1	$+\infty$
$f'(x)$	‖	+
$f(x)$	$-\frac{1}{2}$ ↗	1

$$\lim_{x\to-\infty} f = \lim_{x\to-\infty} f \frac{x}{2\sqrt{x^2\left(1+\frac{1}{x^2}\right)}} = \lim_{x\to-\infty} f = \frac{x}{2|x|\sqrt{1+\frac{1}{x^2}}}$$